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High-Sensitivity Capacitance Method for Measuring Thermal Diffusivity and Thermal Expansion: Results on Aluminum and Copper

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Thermal diffusivity and thermal expansion in high-conducting solids can be measured by means of a capacitance method, which turns out to be simple, reliable, and accurate and yields the first property with an accuracy of ~1% and the second one with an accuracy of ~2%. Preliminary results, which are consistent with the literature, have been obtained on pure aluminum (99.999%) and on commercial copper, both at near room temperature.

KEY WORDS: capacitance method; thermal diffusivity; thermal expansion.

1. INTRODUCTION

The knowledge of thermal diffusivity (α) and of thermal expansion coefficient (β) is fundamental for characterizing the behavior of materials subjected to thermal stress. Consequently it is required in basic and in advanced technology, and particularly in aeronautics, where the design of mechanical components working under severe stress conditions and high thermal gradients requires an accurate determination of the above parameters.

In a recent publication [1] we presented briefly a new experimental method which allows the simultaneous measurement of α and β in high-conducting materials: we note that the high accuracy (~1%) by which α can be obtained is important by itself, as a new contribution to the field of thermal diffusivity measurements.

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In the present paper we discuss in detail the features of the method and apply it to measurements of α and β for both pure aluminum and commercial copper, obtaining values consistent with the literature.

Let us first recall that measurements of thermal diffusivity in solids [2] are generally affected by errors due to uncontrolled heat losses connected with: (1) radiation effects through the surface, (2) thermocouple leads inserted into the specimen, (3) mechanical support of the specimen, and (4) lack of correspondence between the actual heat source $Q(t)$ employed to heat the specimen and the theoretical source introduced into the diffusion equation [3–15].

Finally, in measurements employing the flash method [16–18] one has to consider two other sources of error: (5) the inadequacy of Fick's law to treat the high-temperature gradients near the irradiated area and (6) the nonuniform intensity distribution of the heat source (that is, of the beam irradiating the specimen) [5–12, 16–18].

While the errors due to point 1 are negligible for metals in vacuum below room temperature, those due to point 2 are always present and become especially important at low temperatures, because in this case heat is conveyed to the specimen from the exterior of the vacuum box where the specimen is contained. These errors could be partially avoided by employing radiometric techniques [19, 20] to measure the temperature at some point on the surface. However, such techniques [19] are fully satisfactory only at high temperatures; they have a poor sensitivity at room temperature and become completely inadequate near absolute zero. The four remaining sources of are never eliminated in standard methods and their magnitude cannot be easily estimated. In particular, we stress the fact that the only way to eliminate the problems caused by point 4 would involve measuring the time behavior of the heat source, as proposed in this paper.

2. PRINCIPLES OF THE EXPERIMENTAL METHOD

We present a method for high-conducting materials where the heat source is provided by two parallel rings (D_1 and D_2 in Fig. 1), made of Chromel and Alumel, respectively, and tightened around the middle section of a cylindrical specimen; the two rings are separated by a gap of 0.01 mm. Each ring is pressed against the specimen by the elastic stresses due to its inner diameter, which is slightly smaller than the diameter of the specimen ($2R$). A thin radial cut allows the ring to deform following the change of R due to thermal expansion, without producing excessive constraint on the specimen itself. The cross section of each ring presents, at the contact with the specimen, an edge-shaped profile, the width a of each edge being 0.25 mm. The Joule heat generated by a resistive wire (J) surrounding the rings is transferred by conduction to the specimen and therefore the system of the two rings acts like a heat source. But at the same time, it is a thermocouple by which the temperature of the specimen area in contact with the rings can be easily recorded as a function of time. This represents information from which the heat source $Q(t)$ can be derived, and consequently measured, by numerical solution of an integral equation [see Eq. (8)]. In this way, we eliminate the errors involved in point 4. The small value of a ensures that the temperature of the specimen surface is really uniform in the entire area where it provides contact between the two

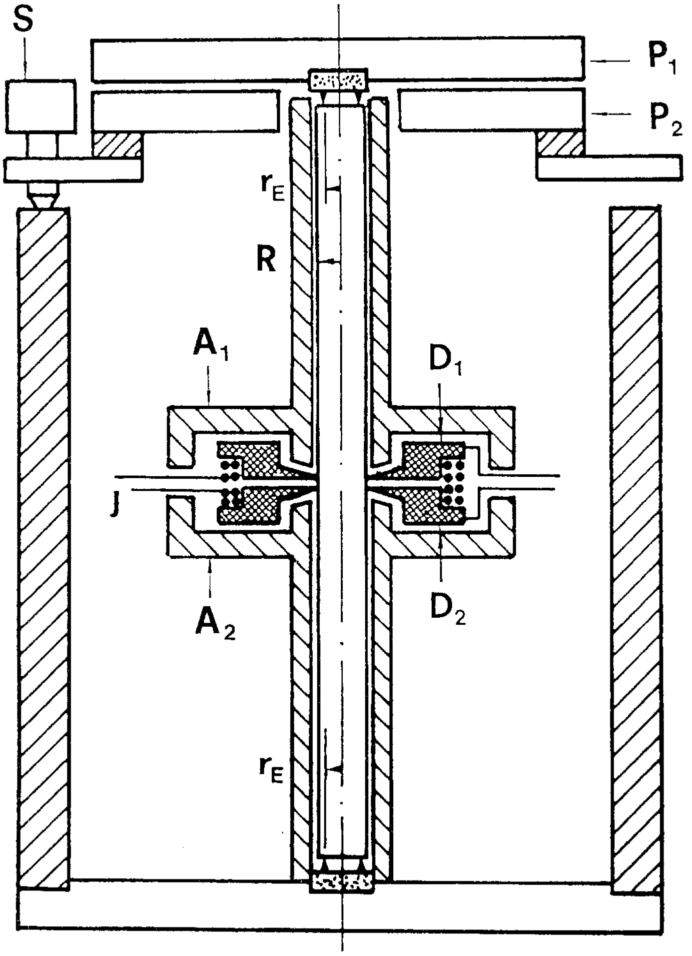

Fig. 1. Schematic diagram of the apparatus. Note that the shields A_1 and A_2 are made of thermal insulating material (plexiglass).

elements of the thermocouple. Consequently, the error introduced by the thermoelectric power of the specimen at the hot junction is absolutely negligible.

Errors due to point 2 are eliminated through the use of a capacitive system where one plate of the capacitor consists of a 10-cm-diameter disk of fused silica (P_1) lying on the upper end of the specimen (see Fig. 1); the second plate is an annular fused silica disk (P_2) surrounding the specimen and placed by three leveling screws (S) on a fused silica support.

By suitable adjustment of the screws one obtains a system of two parallel plates with a gap of the order of 0.05 mm. If the two adjacent surfaces of the disks are coated by a conductive film, this becomes a high-sensitivity detector of any change in length of the specimen. In particular, the temperature field produced by the heat source $Q(t)$ gives rise to dilation effects which can be accurately measured. In this way, the capacitance change as a function of time represents additional information from which the thermal diffusivity can be derived.

It should be pointed out that the two ends of the specimen are separated from the upper plate of the capacitor and from the fused silica base of the apparatus, respectively, by three fused silica pins at 120° at a distance r_E from the axis. Owing to the small contact area between the specimen and each pin (which is essentially a point) and to the low thermal conductivity of fused silica, the uncontrolled heat losses through the ends become completely negligible. In this way, the errors due to point 2 are eliminated because in this method no lead is directly attached to the specimen, and no appreciable heat transfer is introduced by the capacitive system. This feature also obviates the need to consider point 3 since the mechanical support is provided by the fused silica pins. At the same time, one eliminates the errors due to point 5 (because heat is slowly conveyed by conduction through the rings, giving rise to small gradients) and to point 6 (because the width of the annular area is so small that the heat source can be assumed to be uniform in this area).

A few words are now necessary on the technical problems we had to solve in setting up the measuring apparatus.

The connection between each ring of the central thermocouple and the corresponding electric lead is conveniently obtained by a screw inserted into the ring and made of the same material. The electrical connection between the rings must be achieved through the specimen, and not through spurious contacts between the two adjacent surfaces of the rings. Such contacts can be easily prevented if these surfaces are coated by an electrically insulating film.

The capacitive signal is first brought from the capacitor plates to a connector inserted into the wall of the vacuum box. Two shielded cables outside the box lead from the connector to the electric measuring system, which in our apparatus consisted of a GenRad capacitance bridge, and an ITHACO lock-in amplifier. The amplified signal was finally sent to a $Y(t)$ recorder (see Fig. 2).

The sensitivity of the capacitive system was so high that a change in the specimen length of the order of 0.1 nm was easily detected. However, in our measurements at room temperature, such a high sensitivity was never used; it should be invoked only for low-temperature measurements.

Care must be taken to avoid spurious torque applied to the upper disk of the capacitor by the electric lead. This torque can be generally avoided

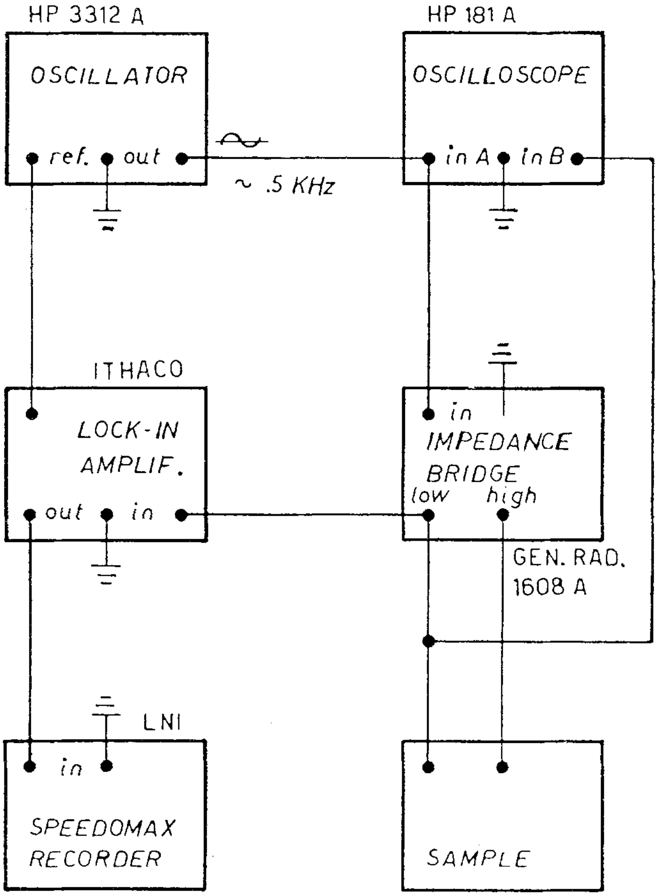

Fig. 2. Block diagram of the instrumentation for high-sensitivity capacitance measurements.

by fixing the lead to the center of the disk on its upper face, just over the contact area between the fused silica pins and the sample. Of course, the conductive coating must be distributed over the whole disk, in order to ensure electrical continuity between its two faces. Alternatively two rigid pieces of a nickel wire, each having the form of an inverted right angle, and about 2 cm long, can be fixed to the lateral border of the disk, in symmetrical positions with respect to its center; the free end of one piece is immersed in a small vessel containing mercury, from which the signal can be brought to the measuring apparatus without any mechanical perturbation to the disk. This method was followed in our measurements at room temperature.

Summarizing, we present a method which is valid at any temperature but is of special interest in the low-temperature range. It is true that as the temperature approaches absolute zero, the expansion coefficient β goes to zero and consequently the capacitive signal is correspondingly reduced. However, the sensitivity of a capacitance system is so high that we estimate the method could be used even for values of β of the order of $10^{-7}\,\mathrm{K}^{-1}$ (which, for typical metals, corresponds to temperatures of a few kelvins). In this case, for a specimen 15 cm long, a temperature change of 1 K would give rise to a change in length of 15 nm, about two orders of magnitude higher than the minimum displacement detectable by our instrument [21, 22].

3. MATHEMATICAL FRAMEWORK AND EXPERIMENT

For a cylindrical specimen of height $2l$ and radius R, where we introduce cylindrical coordinates (r, z, φ) with the z axis along the axis of the specimen and the origin at the center of its middle section, we obtain the temperature field $\theta(r, z, t)$ by solving the heat diffusion equation

$$\nabla^2\theta - \alpha^{-1}\,\partial\theta/\partial t = -k^{-1}q(r, z, t) \tag{1}$$

where α is the diffusivity, k the thermal conductivity, and $q(r, z, t) = Q(t)\,\delta(r-R)$ the heat source, localized in the region $-a \leqslant z \leqslant a$. For a we choose the width of the edge of each ring forming the central thermocouple ($a = 0.25$ mm). At $t = 0$ the specimen is assumed at the uniform temperature $\theta = 0$ (in thermodynamical equilibrium with the environment).

Equation (1) can be easily solved under general boundary conditions of the form $-k\,\partial\theta/\partial n = h\theta$, where $\partial/\partial n$ means the normal derivative at the surface of the specimen, and coefficient h accounts for radiative effects. The solution may be written as

$$\theta(r, z, t) = \int_0^t \Theta(r, z, t-t')\,S(t')\,dt' \tag{2}$$

where

$$S(t) = 4\pi a\alpha Q(t)/klR \tag{3}$$

Denoting by $\{x_s\}$ and $\{y_p\}$ the infinite sets of roots of the trascendental equations

$$yJ_1(y) + Rh/kJ_0(y) = 0 \tag{4}$$

$$x \sin x - hl/k \cos x = 0 \tag{5}$$

and putting $\hat{x}_s = x_s/l$ and $\hat{y}_p = y_p/R$, one has

$$\Theta(r, z, t) = \sum_{sp} \{f_{sp}(r, z) \exp[-\alpha(\hat{x}_s^2 + \hat{y}_p^2)t]\} \tag{6}$$

where

$$\begin{aligned} f_{sp}(r, z) = {} & [\sin(\hat{x}_s a)/\hat{x}_s a] \cos(\hat{x}_s z)\, J_0(\hat{y}_p r)\, J_0(y_p) \\ & \times \left[(1 + \sin(2x_s)/2x_s) \int_0^1 \xi J_0^2(y_p \xi)\, d\xi \right]^{-1} \end{aligned} \tag{7}$$

The source function $S(t)$ can be obtained by a numerical solution of the integral equation

$$\theta_1(t) = \int_0^t \Theta(R, 0, t - t')\, S(t')\, dt' \tag{8}$$

where $\theta_1(t) = \theta(R, 0, t)$ is the temperature recorded by the two-disk thermocouple. The solution of Eq. (8) is obtained for a given arbitrary choice of α and therefore is denoted $S_\alpha(t)$. To see how this function can be used to obtain the true value of α it is necessary to discuss the change of length of the specimen corresponding to the temperature distribution described by $\theta(r, z, t)$. This is given by

$$\Delta(r, t) = \beta \int_{-l}^{l} \theta(r, z, t)\, dz \tag{9}$$

where $\theta(r, z, t)$ is taken from Eq. (2) and β is the linear thermal expansion coefficient.

A numerical study of the solutions of the thermoelastic equations [23, 24] shows that these are in full agreement with the simple expression in Eq. (9) for any temperature distribution depending on z, but not on the radial coordinate (see the Appendix). This assumption is widely verified in metallic specimens, where the high value of the diffusivity ensures, for a given z, a uniform temperature profile within one part in 10^4. Under these conditions, the dilation given by Eq. (9) is independent of r and for this reason we simply write $\Delta(t)$.

On the other hand, we also assume a nonlinear relation of the form

$$\Delta = \rho V + \sigma V^2 \tag{10}$$

between Δ and the measured signal V due to the capacitance change.

In principle, for a given initial value of the capacitance, C_0, the parameters ρ and σ can be determined through an independent experiment by measuring the capacitive signals corresponding to two different values of Δ, imposed by turning the leveling screws. In practice, we found a more convenient procedure, allowing the simultaneous determination of ρ and σ through the analysis of the curves $\Delta(t)$ and $V(t)$ corresponding to the heating of the specimen. This procedure will be discussed later.

From Eqs. (2) and (9) we have the theoretical change of length

$$\Delta^{\text{th}}(t) = \beta \int_0^t \Phi(t-t')\, S_\alpha(t')\, dt' \tag{11}$$

where

$$\Phi(t) = \sum_{sp} \{ g_{sp}(r) \exp[-\alpha(\hat{x}_s^2 + \hat{y}_p^2)t] \} \tag{12}$$

and

$$\begin{aligned} g_{sp}(r) = {} & 2(\sin(\hat{x}_s a)/\hat{x}_s a)[\sin(\hat{x}_s l)/\hat{x}_s]\, J_0(\hat{y}_p r)\, J_0(y_p) \\ & \times \left[(1 + \sin(2x_s)/2x_s) \int_0^1 \xi J_0^2(y_p \xi)\, d\xi \right]^{-1} \end{aligned} \tag{13}$$

Our procedure now is to introduce into Eq. (11) the function $S_\alpha(t)$ deduced from Eq. (8). The theoretical change of length $\Delta^{\text{th}}(t)$ must be fitted to the experimental function given by Eq. (10), where $V(t)$ is now the signal recorded during the heating of the specimen.

The true value of α can be therefore obtained through a least-squares analysis, by minimizing the function

$$\delta(\alpha) = \sum_i [\rho V(t_i) + \sigma V^2(t_i) - \Delta^{\text{th}}(t_i)]^2 \tag{14}$$

where the sum is extended to the N times t_i taken (at regular intervals) through the total time interval of the measurement.

The present paper is essentially concerned with a preliminary set of measurements, performed in vacuum on metallic specimens at room temperature to check the reliability of the method. At this temperature, when the specimen is made of a high-conducting material, we can neglect any heat exchange due to radiation effects even during a considerable time interval, and correspondingly put $h=0$ in Eqs. (2)–(5). Although in this case, once ρ and σ are known, the initial slope of the curves $\theta_1(t)$ and $V(t)$ is sufficient to determine, in a very short time of measurement, the diffusivity α of the specimen, we decided to analyze the whole behavior of the curves corresponding to a total interval of about 500 s, in which the heat source was arbitrarily varied in time. In fact the amount of information we can deduce in this case from experiment is so great that not only α, but also ρ and σ can be obtained from the same pair of curves. In these circumstances the system does not require any previous calibration and provides a powerful and precise method to determine thermal diffusivity. In more precise terms, putting $\rho/\beta=\lambda$, $\sigma/\beta=\mu$, we can minimize, instead of Eq. (14), the expression

$$\delta(\alpha)=\sum_i \left[\lambda V_i+\mu V_i^2-\int_0^{t_i} \Phi(t_i-t')\,S_\alpha(t')\,dt'\right]^2 \tag{15}$$

to be considered as a function of the three variables α, λ, and μ. The minimization with respect to λ and μ can be performed analytically and gives

$$\lambda=\left[\left(\sum_i V_i^4\right)\left(\sum_j V_j I_j\right)-\left(\sum_i V_i^3\right)\left(\sum_j V_j^2 I_j\right)\right]\Big/D \tag{16}$$

$$\mu=\left[\left(\sum_i V_i^2\right)\left(\sum_j V_j^2 I_j\right)-\left(\sum_i V_i^3\right)\left(\sum_j V_j I_j\right)\right]\Big/D \tag{17}$$

where

$$D=\left(\sum_i V_i^2\right)\left(\sum_j V_j^4\right)-\left(\sum_i V_i^3\right)^2$$

and I_i stands for the integral appearing in Eq. (15). These expressions, depending on α, must be substituted into Eq. (15) in order to perform the required numerical minimization with respect to this parameter. We applied the above procedure to a specimen of pure aluminum, with $2l=15$ cm, $R=0.5$ cm; the curves $\theta_1(t)$ and $V(t)$ obtained for an initial capacitance of 385 pF are reproduced in Fig. 3. Table I gives the values obtained for δ from Eq. (15) as a function of α. As may be seen, the minimum is well defined and corresponds to $\alpha=0.87\times10^{-4}\,\mathrm{m^2\cdot s^{-1}}$, which is nearly coincident with the value of $0.86\times10^{-4}\,\mathrm{m^2\cdot s^{-1}}$ given in Ref. 25.

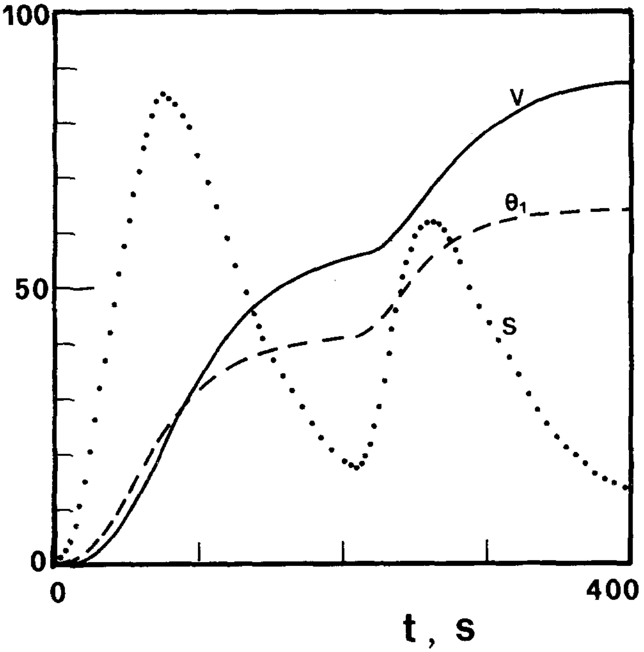

Fig. 3. Heating curves for a specimen of pure aluminum: θ_1 represents the temperature recorded by the two-disk source thermocouple, $V(t)$ the capacitive signal, and $S(t)$ the heat source function as obtained by solving Eq. (8). The y-axis scale is arbitrary.

This result proves the reliability of the method for measurements of thermal diffusivity. The source function as deduced from Eq. (8) and corresponding to the extremal value of α is reproduced in Fig. 3. The usefulness of the method, however, is not confined to a determination of α. By substituting the extremal value of α into Eqs. (16) and (17) one also obtains the relative thermal expansion coefficient of the material under study with respect to a reference material to which the method has been previously applied. In more precise terms, if λ_1 and λ_2 are the resulting values of λ for specimens 1 and 2, respectively, one simply has

$$\lambda_1/\lambda_2=\beta_2/\beta_1 \tag{18}$$

Table I. Values of the Square Sum $\bar{\delta}$ (in Arbitrary Units) as a Function of α (in $10^{-4}\,\mathrm{m}^2\cdot\mathrm{s}^{-1}$) for Aluminum; The Minimum is Well Defined at $\alpha=0.87\times10^{-4}\,\mathrm{m}^2\cdot\mathrm{s}^{-1}$

α	0.84	0.85	0.86	0.87	0.88	0.89	0.90
$\bar{\delta}$	3.19	2.78	2.52	2.38	2.52	2.77	3.14

provided the geometrical configuration of the plates is the same in the two corresponding measurements. In fact, only in this case can one assess the invariance of ρ and σ and, hence, the validity of Eq. (18). The reproducibility of a given configuration of the plates is dependent on their parallelism and on the resulting value of capacitance, C_0. If parallelism is ensured, two experiments with different specimens for which the initial value of C_0 is the same obviously refer to the same pair of values for ρ and σ. The analysis of several heating curves referring to the same specimen and to the same value of initial capacitance C_0, but corresponding to different mountings of the capacitor plates, showed that the value of λ, and consequently of ρ, is reproducible to within ~2% if the following procedure to obtain a good parallelism is adopted: once the upper plate of the capacitor has been put on the tripod mounted on the upper end of the specimen, the leveling screws are adjusted in such a way that, starting from the configuration under study, an equal number of turns imposed independently to each screw produces the same capacitive signal within an uncertainty of ~10%. Using a specimen of pure aluminum, we investigated the consistency of the

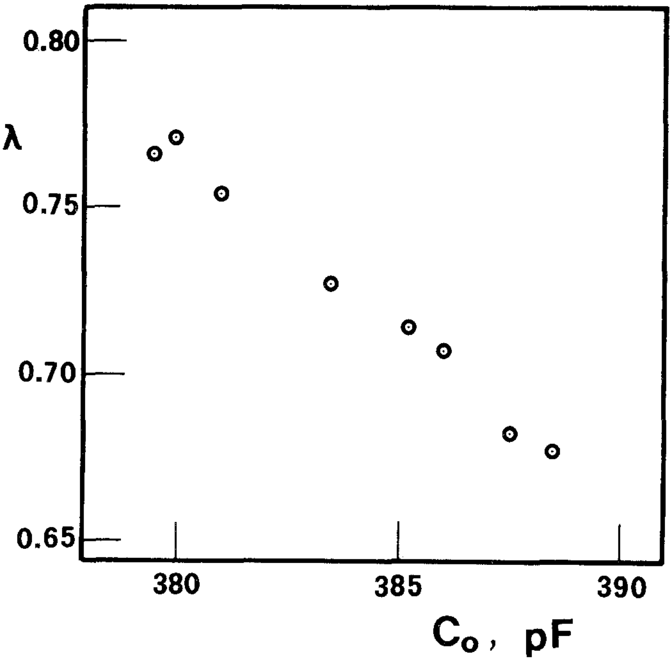

Fig. 4. Values of the parameter λ, as obtained for a sample of Al in different experiments at different initial capacitances C_0. The y-axis scale is arbitrary. Note the quasilinearity of λ with respect to the reference capacitance C_0.

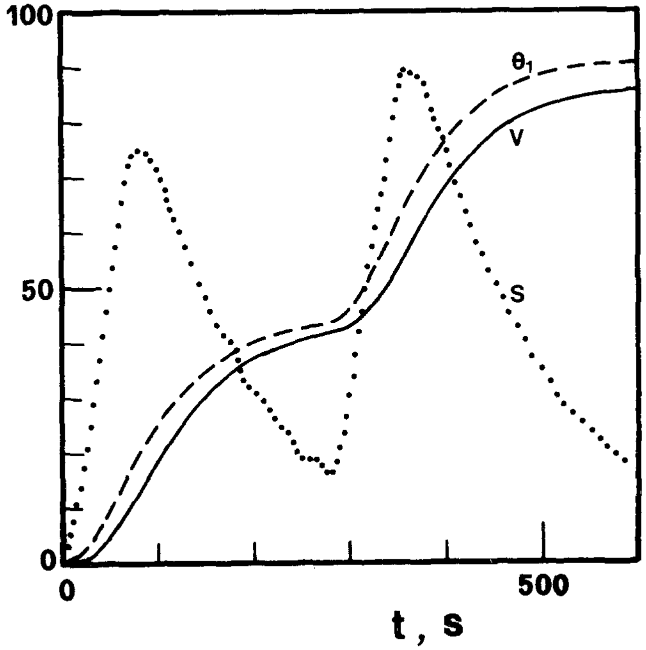

Fig. 5. Heating curves for a specimen of copper: θ_1 represents the temperature recorded by the two-disk source thermocouple, $V(t)$ the capacitive signal, and $S(t)$ the heat source function as obtained by solving Eq. (8). The y-axis scale is arbitrary.

above mounting procedure. For each configuration obtained in this way, we measured the initial capacitance C_0 and heated the specimen. The analysis of the heating curves yielded values of λ which are plotted as a function of the measured value of C_0 in Fig. 4; we deduced a linear correlation between these two parameters, accurate to within ~2%. It was more difficult to establish a correlation between μ and C_0. We could say only that in the interval between 380 and 390 pF, μ was approximately constant, within an uncertainty of about ~20%. Such a rough estimate was, however, sufficient for our purposes, owing to the small magnitude of the nonlinear term.

Table II. Values of the Square Sum $\bar{\delta}$ (in Arbitrary Units) as a Function of α (in $10^{-4}\,m^2 \cdot s^{-1}$) for Copper; The Minimum is Well Defined at $\alpha = 0.93 \times 10^{-4}\,m^2 \cdot s^{-1}$

α	0.90	0.91	0.92	0.93	0.94	0.95	0.96
$\bar{\delta}$	3.29	2.88	2.62	2.52	2.58	2.72	3.02

Table III. Values of the Three Parameters α (in $10^{-4}\,\mathrm{m}^2 \cdot \mathrm{s}^{-1}$), λ, and μ (in Arbitrary Units) Obtained by Minimizing Eq. (15) for Aluminum and Copper; The Measurements Have Been Performed at the Same Capacitance C_0 (385 pF)

	Al	Cu
α	0.87	0.93
λ	0.71	0.98
$\mu\,(10^{-4})$	5	6

In a second experiment, where we used a cylindrical specimen of commercial copper (UNI-5649), we adopted the same mounting procedure as before and measured the capacitance of the resulting configuration. Heating the specimen, we obtained the pair of curves presented in Fig. 5, from which we deduced the well-defined minimum displayed in Table II and the extremal values of α, λ, and μ given in the third column in Table III (the second column provides the corresponding values obtained for aluminum at the same capacitance).

The value of α obtained for our specimen of commercial copper was checked by an independent experiment in which the upper plate of the capacitor was taken out and the temperature $\theta_2(t)$ of the upper face of the specimen was recorded by means of a small thermocouple attached to the center of the end face [26]. By introducing the source function deduced from the two-disk thermocouple into Eq. (2) calculated for $r = 0$, $z = l$, and imposing the best fit to the experimental function $\theta_2(t)$, we deduced $\alpha = 0.93 \times 10^{-4}\,\mathrm{m}^2 \cdot \mathrm{s}^{-1}$, in full agreement with the value in Table III. Moreover, from Table III we obtained $\lambda_{\mathrm{Al}}/\lambda_{\mathrm{Cu}} = 0.727$, again in full agreement with the ratio of the expansion coefficients $\beta_{\mathrm{Cu}}/\beta_{\mathrm{Al}} = 0.724$, as deduced from the data of Wallace [27].

To test the significance of these results, we repeated our measurements with different initial values of the capacitance C_0. We found values of λ and μ slightly dependent on C_0, but the resulting value of α was invariably reproduced within ~1 %.

4. CONCLUSION

Although more extensive measurements on many specimens should be made, from the few data presented in this paper it is already possible to assess the advantages of an experimental method which, in principle, is not restricted to any specific temperature range but becomes of special importance at low temperatures, where it eliminates the problem of heat exchange through the thermocouple leads or of the poor sensitivity of radiometric detectors to measure the surface temperature of the specimen. Finally, we emphasize the practical use of a method which allows, in the frame of a single measurement, the simultaneous determination of two independent parameters such as the thermal diffusivity and the thermal expansion coefficient. This aspect should be carefully considered by industrial laboratories where systematic characterizations of materials are made, with consequent problems of minimizing the measurement times.

APPENDIX

To discuss the approximation inherent in the integral of Eq. (9) one has, in principle, to solve the equation of thermoelasticity:

$$\nabla^2\mathbf{u} + 1/(1-2\nu)\,\text{grad div}\,\mathbf{u} - 2/(1-2\nu)\beta\,\text{grad}\,\theta = 0 \tag{A1}$$

where $\mathbf{u}$ is the displacement field, β the thermal expansion coefficient, θ the temperature field, and ν Poisson's ratio.

We solved this equation for a cylindrical specimen with length $2l$, diameter $2R$, subject to a uniform heat source in the region $-a \leqslant z \leqslant a$ and $r = R$. Note that such a configuration is the actual one used during our experiment. The source was assumed to be

$$S(t) \propto t \exp(-\gamma t) \tag{A2}$$

with $\gamma = 0.015$; this function simulates the real behavior of a typical heat source used in our experiments. First, we solved the Fourier equation to obtain the temperature field $\theta(r, z, t)$ in a specimen of aluminum with

Table AI. Comparison Between $\Delta(R, t)$ as Given by Eq. (9) and the True Displacement Field at $r = R$, $z = l$ for a Cylindrical Specimen with a Temperature Distribution Depending on r, z, and t (High Diffusivity: $\alpha = 0.9 \times 10^{-4}\ \text{m}^2 \cdot \text{s}^{-1}$)

t(s)	10	40	80	120	160	200
$\Delta(R, t)$	0.00320	0.03802	0.10468	0.16578	0.21215	0.24403
$u_z(R, l, t)$	0.00319	0.03797	0.10463	0.16573	0.21211	0.24401

Table AII. Comparison Between $\Delta(R, t)$ as Given by Eq. (9) and the True Displacement Field at $r = R$, $z = l$, for a Cylindrical Specimen with a Temperature Distribution Depending on r, z, and t (Low Diffusivity: $\alpha = 1.0 \times 10^{-7}\,\mathrm{m^2 \cdot s^{-1}}$)

t(s)	10	40	80	120	160	200
$\Delta(R, t)$	0.03156	0.20008	0.40758	0.55499	0.65259	0.71515
$u_z(R, l, t)$	0.00831	0.09940	0.27510	0.43802	0.56392	0.65305

$2l = 15$ cm, $2R = 1$ cm, $a = 1$ mm, and $\alpha = 10^{-4}\,\mathrm{m^2 \cdot s^{-1}}$, neglecting heat losses through the surface. Second, employing a procedure similar to that given by Kovalenko [24], we solved Eq. (A1) for $\beta = 24 \times 10^{-6}\,\mathrm{K^{-1}}$, $\nu = 0.33$, and obtained the displacement field $\mathbf{u}(r, z, t)$. In Table AI, $u_z(R, l, t)$ is compared to the dilation $\Delta(R, t)$ as given by Eq. (9). As may be seen, the discrepancy is always contained within a few parts in 10^3 for any time duration longer than $\sim R^2/\pi^2\alpha$, which for an aluminum specimen with $R = 0.5$ cm amounts to only 25 ms. However, the discrepancy between $\Delta(R, l, t)$ and $u_z(R, l, t)$ turns out to be important for low-conducting materials (see Table AII). We may compare such results with those obtained by assuming the source uniformly distributed over $0 \leqslant r \leqslant R$ in the same axial region $-a \leqslant z \leqslant a$ (see Table AIII). With the same source function given by Eq. (A2), we found that for low-conducting materials the discrepancy between $\Delta(R, l, t)$ and $u_z(R, l, t)$ is again neglible within a few parts in 10^3. This result is a consequence of the fact that, for our source, the temperature field is independent of r (that is, we are dealing with a one-dimensional Fourier equation).

Hence the dilatometric method can be in principle applied also to low-conducting materials [28], provided the heat source is radically changed, so as to render the temperature field independent of the radial coordinate.

Table AIII. Comparison Between $\Delta(t)$ as Given by Eq. (9) and the True Displacement Field at the Upper Face of a Cylindrical Specimen, for a Temperature Distribution Depending Only on z and t ($\alpha = 1.0 \times 10^{-7}\,\mathrm{m^2 \cdot s^{-1}}$)

t(s)	10	40	80	120	160	200
$\Delta(t)$	0.00415	0.04970	0.13756	0.21902	0.28197	0.32653
$u_z(R, l, t)$	0.00414	0.04965	0.13750	0.21897	0.28193	0.32650

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